Open Loop vs Closed Loop Control System

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1 Introduction

The primary objective is to design and implement a robust drive wheel motor control system for a LEGO MINDSTORM EV3 robot. This therefore serves as a practical application of model based design, spanning system identification, controller synthesis, and hardware validation. Key equipment and tools include:

LEGO EV3 robot A two wheeled platform utilising DC motors as actuators for translation and differential steering.

MATLAB and Simulink Used for "External Simulation", where computer code is compiled and executed directly on the lego robot hardware.

Sensors Motor encoders and a gyro angular rate sensor provide real time data back to Simulink for performance processing and display.

These tools bridge the gap between theoretical modeling to the physical robot and allow for the evaluation of both open-loop and closed-loop control systems against strict stability and steady state error specifications.

2 System Modeling

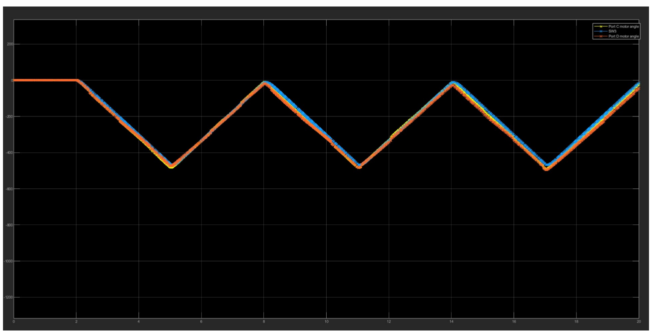
The two motors that drive the wheels can be modeled with a resistor R, inductor L, and an inertial load. By assuming that the inductance is negligible due to a very fast transient, and that all initial conditions are zero, the following equation can be derived:

$$G(s) = \frac{K_m}{s(T_m s + 1)}$$

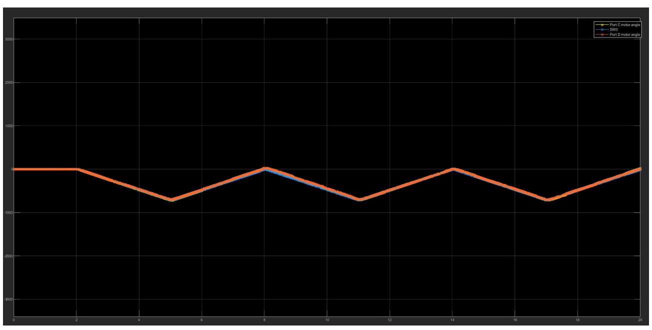
with K_m and T_m being the transfer function parameters that represent the robot's parameters. To simplify the model, friction and damping are assumed to be constant, with no wheel slip. Additionally, the system is treated as perfectly linear, which is not the case in practice.

2.1 Determining Model Parameters

Using trial and error, by setting a value of V_m, then adjusting values until they roughly matched the desired output, suitable values for K_m and T_m were found and the following graphs were recorded:



(a) Open-loop test Vm=20, Km=8, Tm=0.1



(b) Open-loop test Vm=30, Km=8, Tm=0.04

Figure 1: Open-loop system parameter identification testing

It was found that T_m affects the system's response times to a change in input, with higher values tending to lag changes made in the input signal. K_m was found to control the system gain, which would alter the magnitude of the response, with larger values overshooting the desired output and smaller ones not outputting sufficient signal. As seen in Figure 1 above, tests that matched the reference would involve decreasing T_m as V_m increased, while K_m would generally increase when higher values of V_m were used.

3 Controller Design

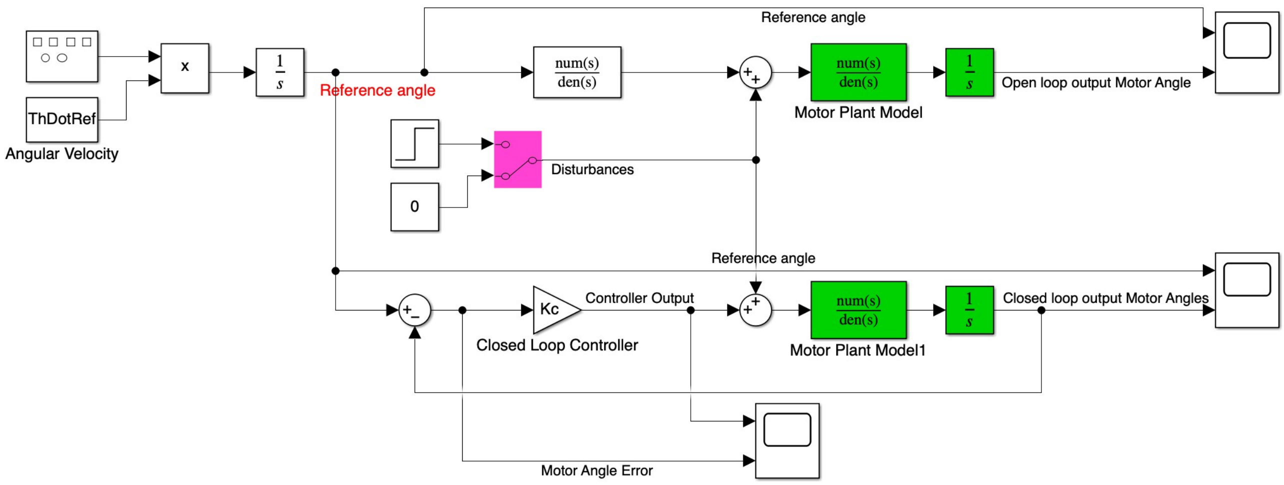


Figure 2: Simulink controller design

3.1 Open-loop Control

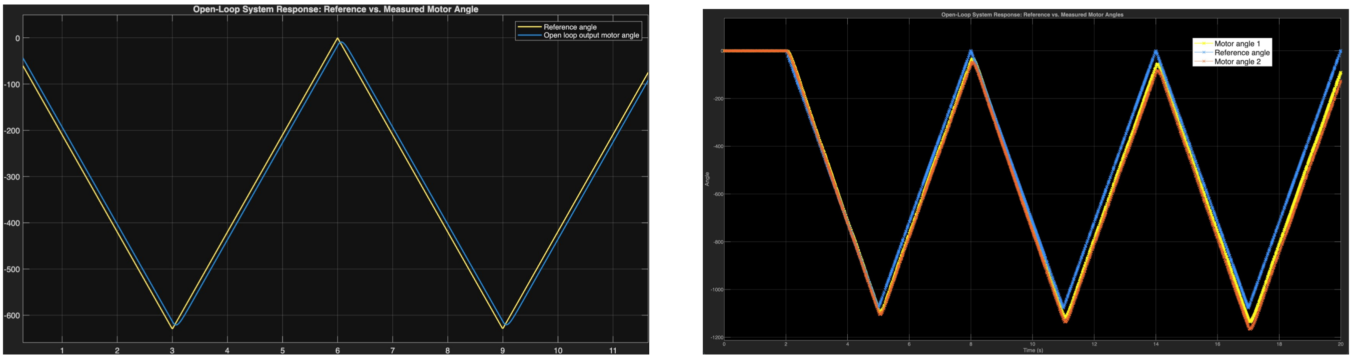
The open-loop controller involves using an inverted linearised plant model around a chosen equilibrium point to mathematically control the system for the desired output. However, to ensure it is physically implementable, the controller must be causal; therefore, a pair of repeated poles at $\frac{-1}{T_o}$ are added. Consequently, the open-loop controller transfer function is:

$$C_{ol}(s) = \frac{s(T_{m0}s + 1)}{K_{m0}(T_o s + 1)^2}$$

with $T_{m0} = 0.04$ and $K_{m0} = 7.5$, set at the equilibrium state found from previous parameter tuning.

Additionally, the filter time constant T_o should be chosen to be small to ensure close tracking of the reference, avoiding larger delays in the reference, and the measured angles. However, making T_o too aggressively small will place the two repeated poles of this second-order LPF at high frequency, making it sensitive to the fast transient and will potentially saturate the controller output. Through experimental trials, T_o was chosen to be 0.01.

Figure 3a shows its good tracking performances when the plant model is ideal.

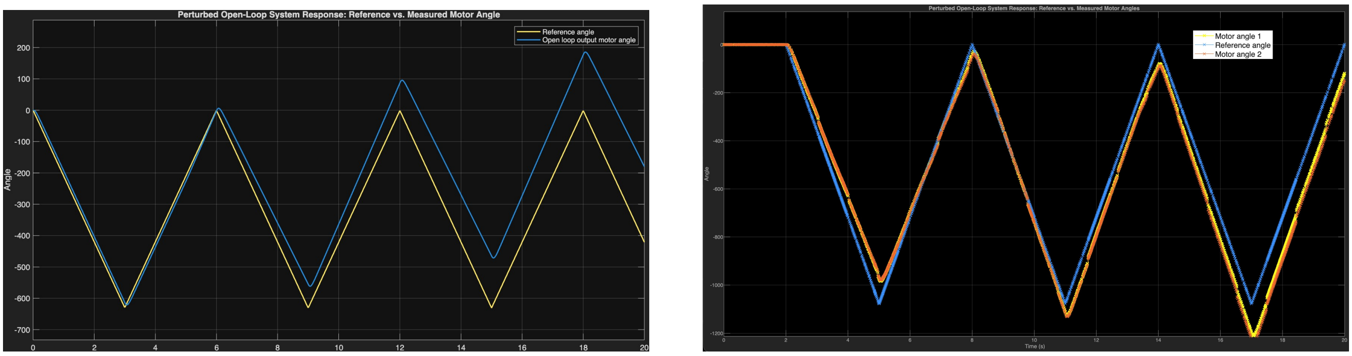


(a) Simulink simulation

(b) Lego kit measurements

Figure 3: Open loop system base response comparison

Conversely, because open loop controller relies solely on the inversed plant model to achieve the desired output, any disturbances to the system or any variations in the plant model in real life will result in poor performance.



(a) Simulink simulation

(b) Lego kit measurements

Figure 4: Perturbed Open loop system base response comparison

3.2 Closed-loop Control

A constant gain feedback controller was considered to combat the aforementioned disadvantages of the open-loop controller. During the design, these three specifications were considered i.e. the system must be stable, the angle error signal is less than 5% and the controller output does not saturate.

3.2.1 Closed-loop System Stability

It can be seen in Figure 5a that no values of Kc will lead to RHP poles, thus the system will be stable for any $Kc > 0$.

3.2.2 Steady State Error

Since the controller transfer function is K_c and the plant model is $\frac{7.5}{s(0.04s+1)}$, the error signal $E(s)$ in steady-state can be found by the Final value theorem.

$$E(s) = R(s) - Y(s) = \frac{1}{1 + K_c G_o(s)} R(s) = \frac{s(T_m s + 1)}{T_m s^2 + s + K_c K_m} \cdot \frac{\dot{\theta}_{ref}}{s^2} \tag{1}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{(T_m s + 1)\dot{\theta}_{ref}}{T_m s^2 + s + K_c K_m} = \frac{\dot{\theta}_{ref}}{K_c K_m} \tag{2}$$

For $\dot{\theta}_{ref} = 210$ and $K_m \approx 7.5$, we require the steady-state error to be less than 5%. That is:

$$\frac{\dot{\theta}_{ref}}{K_c K_m} < 0.05\dot{\theta}_{ref} \implies K_c > \frac{1}{0.05K_m} \approx 2.67 \tag{3}$$

3.2.3 Controller Output Saturation

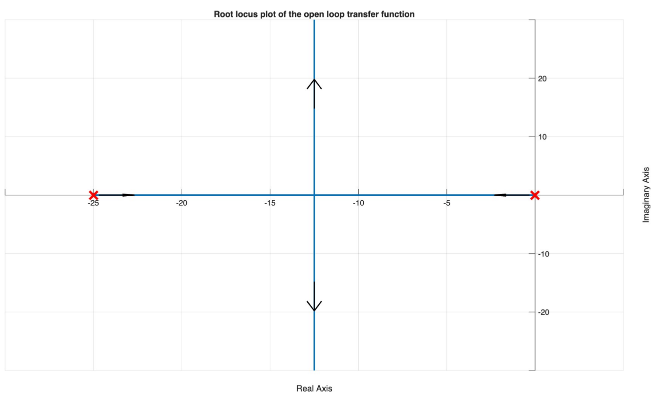
To determine if the controller output saturates for our chosen $Kc = 6$, we analyse the control sensitivity transfer function $S_u(s)$, which relates the reference input to the control effort. The saturation threshold is established by the ratio of the maximum allowable output voltage $V_{m,lim}$ to the magnitude of the reference change V_m. In the provided MATLAB code, the decibel limit is calculated as $20 \log_{10}(V_{m,lim}/V_m)$, which represents the 'ceiling' for the system gain. If the peak magnitude of the Bode plot for $S_u(s)$ exceeds this value, the controller will saturate. The saturation condition is evaluated using the control sensitivity transfer function $S_u(s)$, defined as:

$$S_u(s) = \frac{K_c}{1 + K_c G(s)} = \frac{K_c}{1 + K_c \frac{K_m}{s(T_m s+1)}} \tag{4}$$

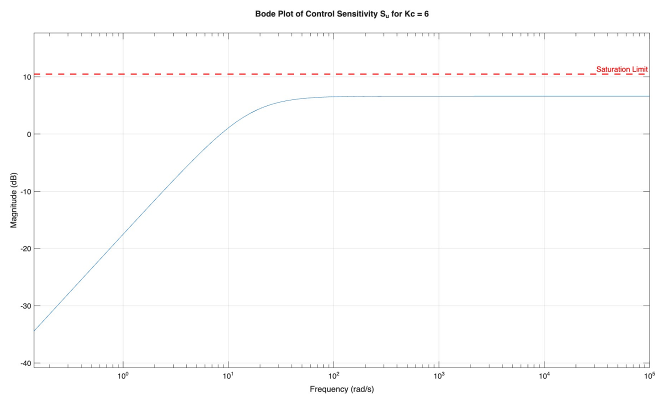
To ensure the controller output does not exceed the hardware limit $V_{m,lim}$ for a given reference magnitude V_m, the peak magnitude of $S_u(s)$ must satisfy:

$$|S_u(j\omega)|_{peak,dB} \leq 20 \log_{10} \frac{V_{m,lim}}{V_m} \tag{5}$$

For the specified parameters where $V_{m,lim} = 100$ and $V_m = 30$, the saturation limit is approximately 10.46 dB. If the peak gain of the controller exceeds this threshold, the control signal will saturate.



(a) Root locus analysis



(b) MATLAB saturation calculation

Figure 5: Closed-loop system analysis

The closed-loop controller design was simulated using the setup in Figure 2, and met specifications well. The results show that our chosen constant gain feedback controller achieved the second and third specifications as seen in Figure 6a and is robust to system disturbances and perturbations as seen in Figure 6b. This will be explored further in the following section.

4 Implementation and Testing

Both controller designs were implemented on the LEGO EV3 robot using Simulink External Simulations. By allowing real time communication between MATLAB/Simulink and the robot during operation, the motor encoder measurements and gyro sensor data can be streamed to Simulink for monitoring and analysis.

Experimental testing was conducted using triangle wave angular position references corresponding to a wheel speed of 210 deg/s and square wave frequency of 1/6 Hz.

4.1 Open-loop Testing

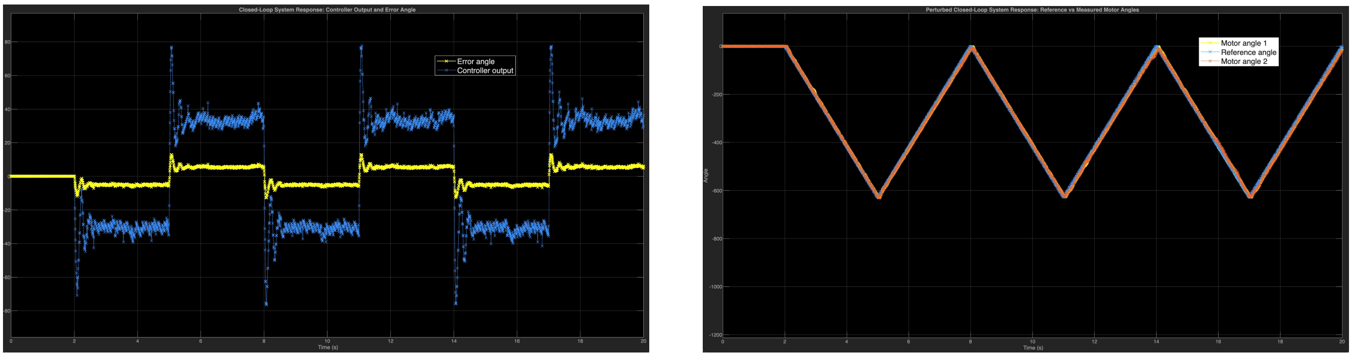
4.1.1 Base Response

The experimental base response of the open loop system in Figure 3b shows acceptable performance under no external influences, closely mirroring the simulated results as shown in Figure 3a. Under normal operating conditions, the measured motor angles closely followed the reference trajectories with relatively small steady state error. However, the error in motor angle with respect to the reference does increase with time as seen in Figure 3b. This is likely due to assumptions made and unmodeled dynamics such as friction, battery voltage variation, and cumulative disturbances in the open-loop system.

4.1.2 Perturbed Response

The open-loop controller shows poor disturbance rejection as shown by the simulation in Figure 4a. Over time, it can be seen that the controller output greatly deviates from the reference signal as a result of external disturbances added to the system. In practice, Figure 4b mirrors the simulation where perturbations such as pushing the robot or disturbing the wheels resulted in persistent errors. This is because the controller did not receive feedback to correct deviations between the desired and measured motor angles. Aggressive disturbances may also result in voltage saturation and greater angle error percentages. These observations demonstrated that open-loop control is highly sensitive to modeling uncertainty and lacks robustness under practical operating conditions.

4.2 Closed-loop Testing

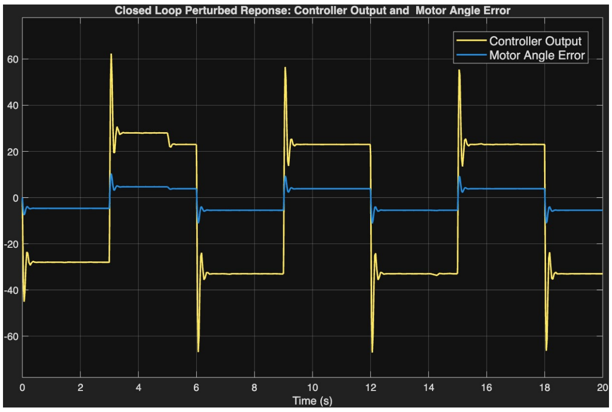


(a) Controller output and error angle

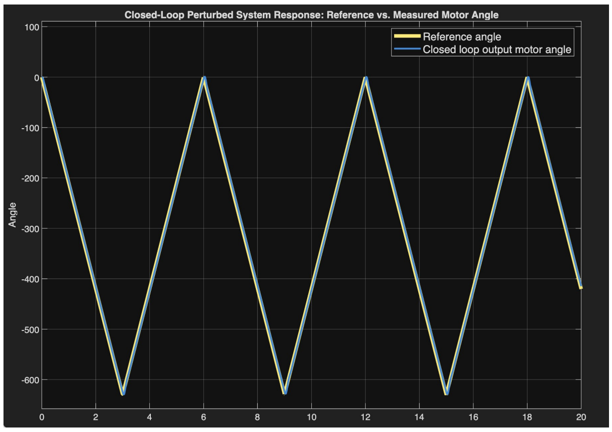
(b) Perturbed closed loop system response

Figure 6: Closed-loop system experimental waveforms

It can be seen in Figure 6b that the closed-loop controller exhibited strong disturbance rejection and achieved stable operation and accurate tracking of the reference position, whereby the measured motor angles closely followed the reference trajectories with minimal steady state error. Compared with the open-loop controller, the feedback controller produced significantly improved robustness and tracking consistency despite variations in motor behaviour and operating conditions (perturbations). This is due to the controller rapidly correcting the resulting positional error by adjusting the motor voltage based on the measured error feedback signal when external disturbances were applied to the system. In contrast, the open-loop controller was unable to compensate for disturbances and exhibited persistent tracking error.



(a) Simulated Controller output and error angle



(b) Perturbed closed loop simulated response

Figure 7: Closed-loop system perturbed simulated waveforms

Although the overall experimental behaviour was consistent with simulation results as shown in Figure 7, the measured response in Figure 6 contained greater noise due to the aforementioned assumptions made, electrical noise, mechanical vibrations, and small asymmetries between the two motors resulting in potentially differing wheel responses, whereas the simulation assumes ideal conditions. Despite these modeling inaccuracies, the closed-loop controller successfully satisfied the key specifications. The system remained stable throughout testing, where bounded inputs (perturbations) resulted in bounded outputs. The steady state tracking error was approximately 3.5%, remaining within the 5% specification and the motor voltage had a maximum peak of 78%, remaining well below the saturation limit of 100% during normal operation as seen in Figure 6a. These results were validated with a substantially greater robustness, improved tracking performance, and better tolerance to disturbances and modeling uncertainty compared with the open-loop controller design.

5 Conclusion

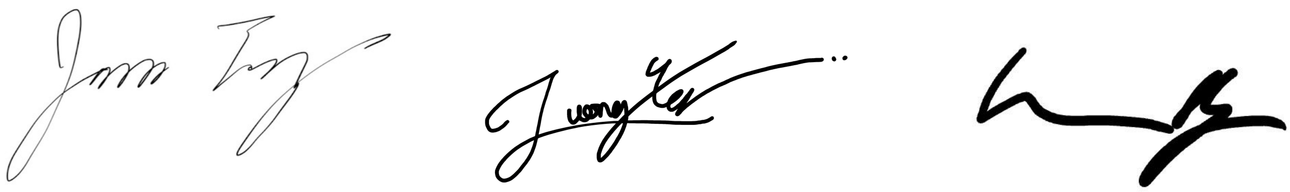
A DC motor transfer function was developed and validated using measured motor response data from MATLAB/Simulink and experimental testing in order to model and control the LEGO EV3 drive wheel motors.

Both open-loop and closed-loop controllers were investigated. Simulations and experimental results show that the open-loop controller provided acceptable tracking only under nominal conditions and was highly sensitive to disturbances and variations in parameters. In comparison, the closed-loop controller achieved improved stability, tracking accuracy, and robustness due to the encoder feedback.

Additionally, both theory and experimental testing showed that the closed-loop controller satisfied the three required performance specifications while maintaining stable operation under varying conditions. With that being said, possible improvements include implementing a PID control to further reduce steady state error and improve transient response. A more accurate motor model which incorporates friction, wheel slip, and variations in battery voltage could also improve both simulation accuracy as well as controller performance. Overall, the project demonstrated the importance of feedback control in improving system robustness and practical control performance.

A Contribution Statements

James Teng	System data collection, Simulink Controller Design, Lego Kit Controller Testing
Tuong Bao Nguyen	Introduction, Testing and Implementation, Conclusion
Ethan Ng	System Modeling, Simulations



(a) James Teng (b) Tuong Bao Nguyen (c) Ethan Ng